*Hydrodynamic attenuation*

The nature of hydrodynamic attenuation was clearly illustrated in a simple way by Sir Geoffrey Taylor in 1939[6]. In Fig. 1 are shown pressure–distance profiles of a shock wave in a fluid at two successive times. The velocity of propagation of a point $Q$ immediately behind the shock front is the local sound velocity, $c$, plus the particle velocity, $u$, with which it is carried along behind the shock. It is readily shown that $u + c > R$, the propagation velocity of the shock itself. As the rarefaction overtakes the shock front, the shock amplitude is diminished. If we assume that the diminution in shock amplitude which occurs when point $Q$ overtakes the shock front is exactly equal to $-\Delta s\, \partial P/\partial x$, where the derivative is evaluated immediately behind the shock front, we can readily determine the rate of decay of the shock. Point $Q$ travels the distance $\Delta s + \Delta X$ in the same time it takes the shock front to travel the distance $\Delta X$, i.e.

$$\Delta t = \frac{\Delta X}{R} = \frac{\Delta X + \Delta s}{u + c}.$$

For $\Delta p = -\Delta s\, \partial p/\partial x$, this gives

$$\frac{\Delta P}{\Delta X} \to \frac{\mathrm{D}P}{\mathrm{D}X} = -\frac{(u + c - R)}{R}\frac{\partial P}{\partial x}. \tag{2.1}$$

The difference $u + c - R$ increases monotonically with the curvature of the Rankine–Hugoniot $p$–$v$ curve, so equation (2.1) shows that shock decay is rapid where the thermodynamic derivative $\partial^2 p/\partial v^2$ is large in the shocked state and $\partial p/\partial x$ is large, i.e. the shock is a sharp spike.

A. J. Harris in 1942 and 1943 derived an exact equation for decay of shock waves in fluids for plane, cylindrical and spherical geometries, assuming that the shock front is discontinuous and flow behind the shock is isentropic[7, 8]. His result for a fluid with an arbitrary equation of state is [8]:

$$R\left(R - u + \frac{c^2}{v}\frac{\mathrm{d}u}{\mathrm{d}p}\right)\frac{\mathrm{D}P}{\mathrm{D}X} = [(R-u)^2 - c^2]\frac{\partial P}{\partial x} - \frac{(n-1)c^2 u(R-u)}{vX} \tag{2.2}$$

where $n = 1$ for plane waves, 2 for cylindrical waves and 3 for spherical waves, $v$ is specific volume, and $X$ is the position of the shock front. When $n = 2$ or 3, the second term on the right provides 'geometric attenuation', which exists even for infinitesimal waves. The first term on the right-hand side of equation (2.2) vanishes like $u^2$, so it is negligible for infinitesimal waves.

Equation (2.2) with $n = 1$ contains terms in addition to those in equation (2.1). In the approximation of equation (2.1), the reflected wave produced when the overtaking rarefaction reaches the shock front is neglected. The amplitude of this reflected wave is of the order $u^3$, so equations (2.1) and (2.2) differ sensibly only for strong shocks.

Maxwell attenuation can be illustrated by a simple relaxing fluid. The relation between pressure and density under isentropic conditions for a non-relaxing fluid is $\mathrm{d}P/\mathrm{d}\rho = c^2$ or $\dot{P} = c^2\dot{\rho}$, where the dot denotes the convective derivative and $c$ is sound velocity. In this case the equations of continuity and motion can be combined to give a pair of equations for

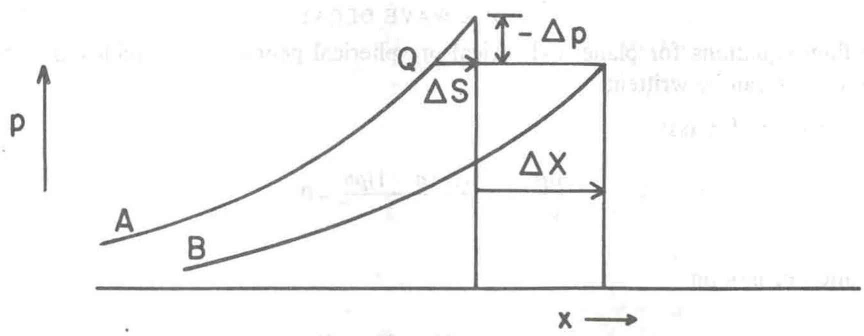


Fig. 1. Decay of a plane shock wave. A and B are wave profiles at times $t$ and $t + \Delta t$, respectively. $\Delta t$ = time required for the point Q to overtake the shock front.

travelling waves[9]. For waves propagating in the $+x$ direction, this equation is

$$\frac{D(u+l)}{Dt} = 0 \tag{2.3}$$

where $D/Dt = \partial t + (u+c)\,\partial/\partial x$, and $l = \int dP/\rho c$. A statement equivalent to equation (2.3) is that $u + l = \text{const.}$ on every $C_+$ characteristic.

Suppose now that every incremental change in pressure in an element of the fluid is followed by relaxation to some equilibrium state, say $P_S(\rho)$. Then the relation between $P$ and $\rho$ has the form

$$\dot{P} = c^2\dot{\rho} - F(P, \rho) \tag{2.4}$$

where $F(P, \rho)$ is the relaxation function. For example, a simple approximation to $F$ might be

$$F = |P - P_s(\rho)|/T \tag{2.5}$$

where $T$ is a constant relaxation time.

When equation (2.4) is combined with the plane flow equations, equation (2.3) is replaced by

$$\frac{D}{Dt}(u+l) = -F/\rho c. \tag{2.6}$$

For small disturbances $l \simeq \Delta P/\rho c$ with $\rho$ and $c$ approximately constant. Moreover, $u \simeq \Delta P/\rho c$, so equation (2.6) becomes

$$D(\Delta P)/Dt \simeq -F/2 \tag{2.7}$$

or

$$D(\Delta P)/Dx \simeq -F/2c \tag{2.8}$$

where $c$ is the propagation velocity of small disturbances. Equation (2.7) is analogous to equations which describe other decay processes, say the decay of a radioactive population, $N$: $dN/dt = -\lambda N$. The difference is that the time derivative in this case is a directional derivative along the path of wave propagation. If one considers a layer of finite thickness, $\Delta x$, the wavefront takes a finite time, $\Delta x/c$, to cross the layer, and during this time the wave amplitude decays an amount $F\Delta x/2c$. However small $\Delta x$ may be, this decay occurs and accumulates from one layer to the next; or, from equation (2.4), if a mass element is out of equilibrium, i.e. $P \neq P_s$, $P$ undergoes a reversible change $c^2\dot{\rho}\Delta t$ in time $\Delta t$, and in addition it changes by $-F\Delta t$. The latter change occurs even if $\dot{\rho} = 0$.

It will be shown in the next section how geometric, hydrodynamic and Maxwell attenuation combine to produce the net decay of a shock wave in a solid whenever pressure in the shock depends on variables other than material density.

## 3. SHOCK WAVE DECAY

The flow equations for plane, cylindrical or spherical geometry in which only one space variable occurs can be written:

Conservation of mass:

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x} + \frac{(n-1)\rho u}{x} = 0. \tag{3.1}$$

Equation of motion:

$$\rho\frac{du}{dt} = -\frac{\partial p_x}{\partial x} - \frac{2(n-1)\tau}{x}. \tag{3.2}$$